The number of cases, the second letter and envelope used (without the first a) is $u_{n-1}-u_{n-2}$.

The number of cases, the third letter and envelope used (without the first having been a or the second b) is $u_{n-1}-2u_{n-2}+u_{n-3}$.

Thus we have successively,

$$u_{n-1},$$
$$u_{n-1}-u_{n-2},$$
$$u_{n-1}-2u_{n-2}+u_{n-3},$$
$$u_{n-1}-3u_{n-2}+3u_{n-3}-u_{n-4},$$
$$\ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots \quad \ldots\ldots$$

The sum of these n lines is

$$S=u_n\left[\frac{1}{1}-\frac{1}{2!}+\frac{1}{3!}-\frac{1}{4!}+\ldots\ldots-\frac{(-1)^n}{n!}\right].$$

C, the required chance, is $1-S/u_n$.

$$\therefore\ C=\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\ldots\ldots+\frac{(-1)^n}{n!}.$$

[NOTE. A solution of problem 61 will be published in the next issue. If any one will send us a solution of problem 65, it will also appear in the next number. ED. F.]

67a. Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.

A witness in court who undertook to recognize the signature of an individual failed four times in succession. What is the probability that he was correct the fifth time? An actual occurance.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let p represent the probability of his failing again the fifth time, then we have,

$$p=\int_0^1 x^5\,dx \div \int_0^1 x^4\,dx=\tfrac{5}{6}.$$

$\therefore$ The probability of his being correct the fifth time is $\frac{1}{6}$.

II. Solution by the PROPOSER.

In the absence of other evidence we are compelled to judge of the credibility of the witness by his present testimony.

If x represents his credibility, the *apriori* probability that he would have testified falsely four times in succession is $(1-x)^4$.

Hence the chance that x had any particular value is $(1-x)^4dx/\int_0^1(1-x)^4dx$ $=5(1-x)^4dx$, and the chance of the event if x has this particular value is x.

Hence the chance of the event through any particular value of x is $5x(1-x)^4dx$ and the chance through all possible values is $\int_0^1 5x(1-x)^4dx=\frac{1}{6}$.

That is, the chances are 5 to 1 against the correctness of his testimony.